Beamlets from stochastic acceleration

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We investigate the dynamics of a realization of the stochastic Fermi acceleration mechanism. The model consists of test particles moving between two oscillating magnetic clouds and differs from the usual Fermi-Ulam model in two ways. (i) Particles can penetrate inside clouds before being reflected. (ii) Particles can radiate a fraction of their energy during the process. Since the Fermi mechanism is at work, particles are stochastically accelerated, even in the presence of the radiated energy. Furthermore, due to a kind of resonance between particles and oscillating clouds, the probability density function of particles is strongly modified, thus generating beams of accelerated particles rather than a translation of the whole distribution function to higher energy. This simple mechanism could account for the presence of beamlets in some space plasma physics situations.

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I. INTRODUCTION

More than half a century ago Fermi [1], in the context of cosmic rays acceleration, proposed a simple model which describes how relativistic charged particles can be accelerated through stochastic encounters with magnetized clouds (see Ref. [2] for a review). The model became rapidly popular and is referred to as a paradigm for further studies in a wide range of different physical systems [3–9]. For example, within a dynamical systems framework, the so-called Fermi-Ulam model (FUM) [10–15] describes the bouncing of a ball between a sinusoidally oscillating wall and a fixed one. This system can generally be written as a two-dimensional map, whose coordinates are the velocity of the ball after a collision with the wall and the phase of the moving wall [16]. A simplified version of the FUM has been proposed in order to reduce computational time [11,17]. In this system the displacement of the wall is ignored and only the momentum exchanged between the ball and the wall is retained. However, this model underestimates the acceleration and further modifications have been done by taking into account the effect of the wall displacement [14]. When both the original and modified FUMs have been run for an ensemble of particles with a well defined distribution function, energization of the whole distribution function has been observed. This means that distribution functions of particles collected at different times can be properly rescaled to the initial one. The energization of the bulk of particles can also be observed in numerical simulations of particles in turbulent [18] and stochastic fields [19].

In variance to stochastic acceleration that apparently gives rise to energization of the bulk, collimated beams of accelerated particles are observed in space plasmas. For example, solar flares are even characterized by the presence of particle beams [20,21] that look to be a different population with respect to the core. Electromagnetic emission in space is generally attributed to beams of accelerated particles [22,23]. Recently, using different satellite crossings, magnetic field-aligned ion beams (called "beamlets") have been detected in the Earth's magnetotail [24–26]. Even if beams of accelerated particles have never been observed within stochastic acceleration mechanisms, it has been conjectured [25,26] that the acceleration mechanisms of beamlets could be due to the interaction between particles and moving magnetic structures, with the aid of some underlying selection mechanism.

In the present paper we show that beamlets could also be produced within a Fermi-like mechanism. This is due to the fact that when a particle penetrates inside a cloud before being reflected, after a certain time it becomes resonant with the cloud. The particle can then be captured within the clouds and this mechanism is able to avoid the complete loss of the energy acquired by the particle. In Sec. II we present the model and in Sec. III we show numerical results. Finally in Sec. IV we briefly outline how a specific model for beamlets in space can eventually be built up from our results.

II. A SIMPLE MODEL FOR STOCHASTIC ACCELERATION

Let us consider a version of the FUM [10] where nonrelativistic charged particles of mass m move between magnetic clouds of mass $M \gg m$, along the x direction. The boundaries of magnetic clouds that correspond to the moving wall in the FUM, are initially placed at positions $s_k X_0$. The index k stands for left (L) and right (R), respectively, and $s_k = \pm 1$ according to the fact that the boundary of the L cloud is placed initially at position $-X_0$, while the boundary of R cloud is placed at X_0. The regions of space $x \leqslant -X_0$ and $x \geqslant X_0$ correspond, respectively, to the L and R cloud. Each boundary moves independently following a given oscillating functional shape described by

$$X_k(t) = s_k X_0 + A_k \sin(\omega_k t + \alpha_k) \tag{1}$$

around the positions X_0. Here ω_k represents the frequency of the oscillating motion, A_k the amplitude of the motion, and α_k the random phase chosen in the range $[0,2\pi]$. The distance $l(t)$ between the clouds is assumed to be greater than zero for all times t. A particle between the clouds moves and collides following the Newtonian laws of motion for a point body. If $V = dX/dt$ and u represent, respectively, the veloci-

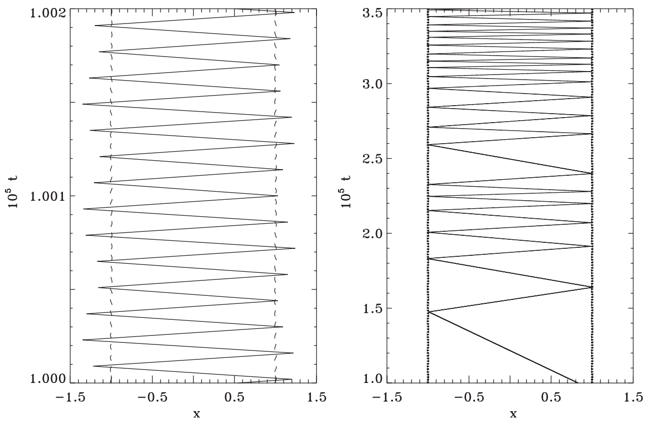

FIG. 1. The space-time behavior of two different test particles moving between the walls (full lines) is showed for two values of b_0, namely, $b_0 = 2\times10^{-5}$ (left panel) and $b_0 = 2\times10^{-7}$ (right panel). The time behavior of the oscillating clouds, being the amplitudes of the motion very low compared to the distance between them, is viewed in this plot as a couple of vertical dotted lines at positions $x = \pm 1$, their oscillating behavior cannot be appreciated. The random penetration inside the clouds is well visible in the left panel, corresponding to the higher value of b_0.

ties of a cloud and of the charge before a collision, after that the particle velocity becomes $-u+2V$. Then the difference of energy before and after a collision $\Delta E = 2m(V^2 - \sigma|u||V|)$ depends on the relative (random) sign $\sigma = uV/|u||V|$ between the speed of the charge and that of the cloud during the collision. Head-on collisions ($\sigma = -1$) increase the energy of the particle by a factor ΔE, while tail-on collisions ($\sigma = 1$) lead to a decreasing in energy.

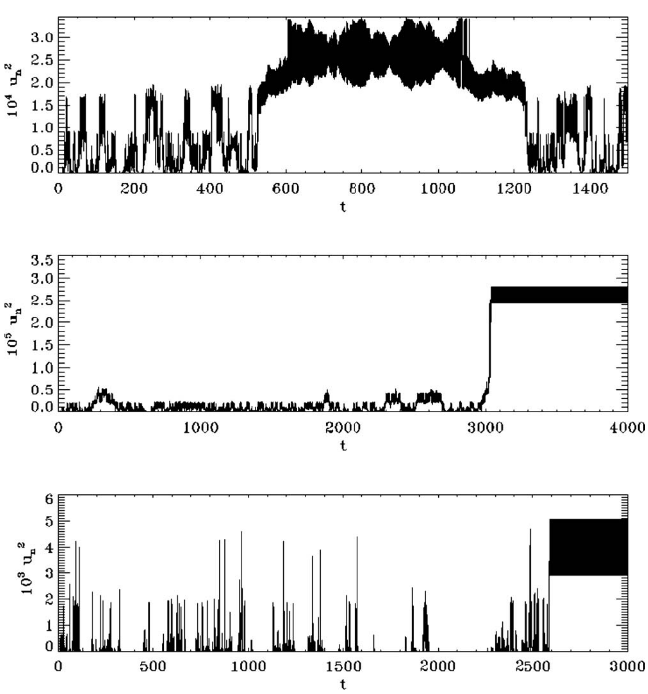

FIG. 2. Velocity square gained by a test particle as a function of time, for a system with $\Omega = 1$. From top to bottom $b_0 = 2\times10^{-7}$, $b_0 = 2\times10^{-6}$, and $b_0 = 2\times10^{-5}$. The value of μ is set to $\mu = 0$.

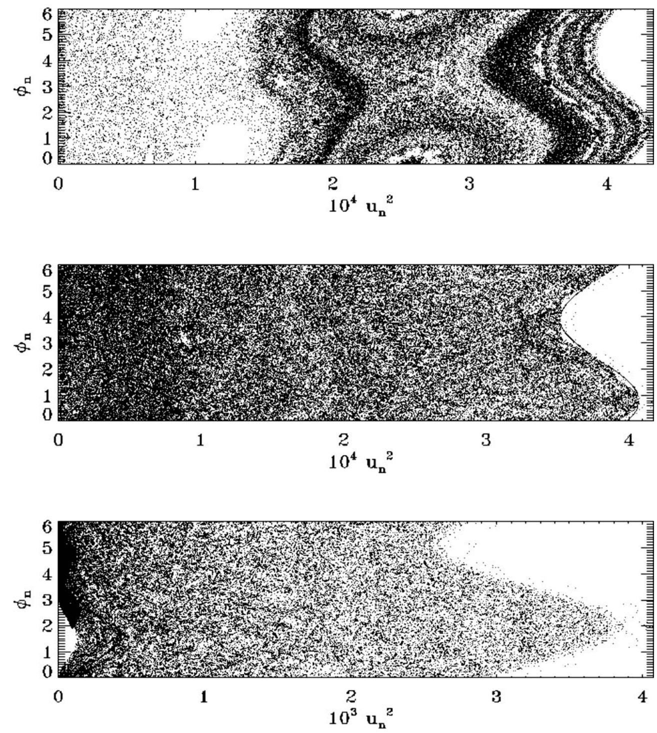

FIG. 3. We show the phase space (u_n^2, ϕ_n) of a test particle for the system with $\Omega = 0$, say one fixed and an oscillating cloud. Panels refer to three different values of b_0, namely, $b_0 = 2\times10^{-7}$ (upper panel), $b_0 = 2\times10^{-6}$ (middle panel), and $b_0 = 2\times10^{-5}$ (lower panel). The value of μ is set to $\mu = 0$.

According to the original idea by Fermi [1], clouds are viewed as regions of space where the magnetic field is concentrated and particle can also penetrate [19]. Instead of describing the dynamics of the model as a nonlinear map [12], we integrate the Newton equation of motion with a variable time step. However, since the dynamics depends on encounters between particles and clouds, we introduce a discrete variable n which counts successive collisions, a collision being defined through the time t_n at which $|X_k(t_n) - x_p(t_n)| < \epsilon$, where x_p is the particle position and $\epsilon \ll 1$. Our choice of integrating Newton laws of motion comes from the fact that, as said before, here we consider a peculiar feature with respect to the classical FUM. Since clouds represent regions of space where the magnetic field is concentrated, a particle does not actually collide with a rigid wall, but when it arrives at the cloud position at time t_n it can penetrate the cloud before being reflected by the action of the Lorenz force. We assume that the particle can penetrate a distance $r_n = B_n u_{n-1}$, where B_n is a random parameter which depends on the magnetic field intensity within each cloud and u_{n-1} is the velocity of particle when it encounters the cloud (see Fig. 1). Inside the cloud particle can radiate energy at the Larmor rate proportional to its squared acceleration [7]

$$\frac{dQ}{dt} \sim -\gamma(u_n^* - u_{n-1})^2, \tag{2}$$

where $u_n^* = -u_{n-1} + 2V_n$ is the velocity of the particle after the collision, without considering the energy losses via radiation

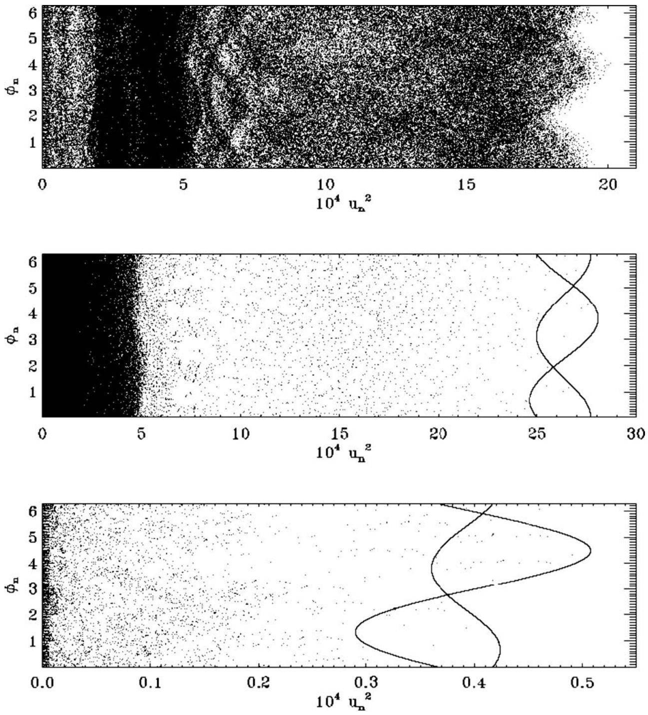

FIG. 4. The same of Fig. 3 for two oscillating clouds with $\Omega=1$.

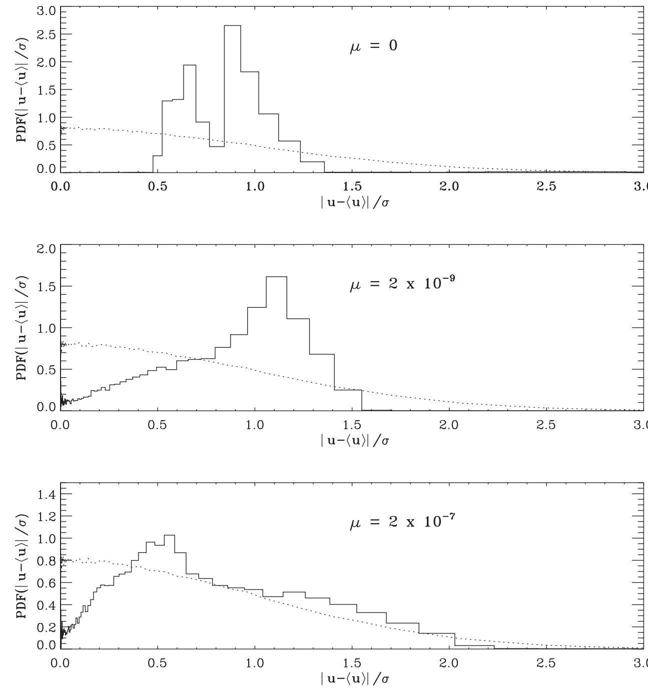

FIG. 5. Probability density functions of the absolute value of standardized velocity fluctuations of an ensemble of 10^6 particles for $\mu=0$ (upper panel), $\mu=2\times10^{-9}$ (middle panel), and $\mu=2\times10^{-7}$ (lower panel). Dotted line refers to the initial Maxwellian distribution, solid line corresponds to the distribution of particles at time $t_{\max}=5\times10^5$. The oscillation frequencies ratio is set to $\Omega=1$.

emission. Finally, taking into account the radiation energy, the particle velocity after the nth collision is given by [7]

$$u_n=\mathrm{sgn}\{u_n^*\}[|u_n^*|-\mu(\gamma)|u_n^*-u_{n-1}|], \tag{3}$$

where μ is a dimensionless free parameter that depends on the amplitude of the radiation loss.

III. NUMERICAL RESULTS

In order to have dimensionless quantities, lengths were normalized to X_0 and times to $1/\omega_L$. This leads to rewrite the equations of motion of the two clouds as

$$X_L(t)=-1+A_L\sin(t+\alpha_L),$$

$$X_R(t)=1+A_R\sin(\Omega t+\alpha_R),$$

where $\Omega=\omega_R/\omega_L$. Particle velocities are normalized to the thermal speed u_{th}, so that $\mu=[2\gamma/(m\omega_R)]\Omega$ and $b_n=[B_n u_{\mathrm{th}}/(X_0\omega_R)]\Omega$. We conjecture that the magnetic field inside clouds is random, so that b_n, which is proportional to the inverse of the magnetic strength, is taken as a random number uniformly distributed in the range $b_n\in[0,b_0]$. We keep the amplitudes of oscillations $A_L=A_R=0.01$ fixed and we present results as Ω, b_0, and μ are varied.

First of all let us consider the dynamics of a single test particle, their initial speed is randomly chosen in the interval $[-1,1]$ and it is injected at a random position between the two clouds. In Fig. 1 we show the trajectory of a test particle between the clouds for two values of the parameter b_0, namely, $b_0=2\times10^{-7}$ and $b_0=2\times10^{-5}$, which shows the different penetrations inside the clouds. In this case $\mu=0$, that is, the particle does not radiate energy. In order to investigate how the value of the penetration inside a cloud influences the particle dynamics, the temporal variation of the kinetic energy u_n^2 has been studied by varying the parameter b_0. In Fig. 2 the time evolution of u_n^2 for three different values of b_0 is displayed. In the upper panel we show the case when b_0 is low enough. We see that the particle acquires energy in the form of bursts at some fixed times. This means that the particle loses all energy acquired. However, we can clearly observe that, for some period of time, the energy acquired by the particle does not decay to zero. In this period the energy seems to be confined, that is, the dynamics is able to avoid the complete loss of energy. This situation is unstable and, after a certain time, the dynamics goes back and the particle will lose energy. However, what is interesting is that by increasing the value of b_0, a different behavior occurs. The cooperative effect of the two oscillating clouds can confine the energy of the particle thus breaking the usual Fermi mechanism. As expected from Fermi acceleration the energy is acquired by the system, but after an initial transient, it remains almost constant with random oscillations around an average value. Energy is never completely lost in successive encounters with clouds and, for relatively higher values of b_0, the energy is confined for all times and the above situation persists forever.

The above difference can be also highlighted by looking at the phase space (u_n^2,ϕ_n), where $\phi_n=\omega_k t_n \mathrm{mod}\ (2\pi$ is the phase of the oscillation of the kth cloud at the collision time, each point representing a single collision. In Fig. 3 the phase

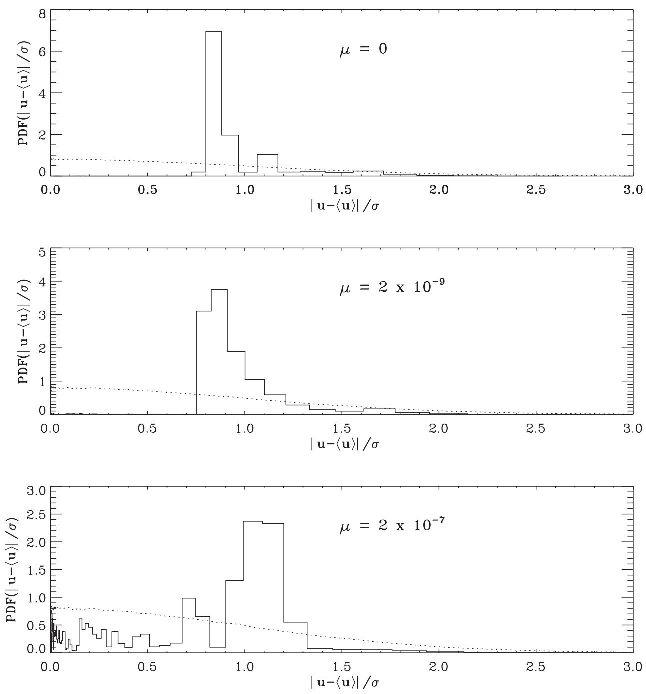

FIG. 6. The same as Fig. 5 for $\Omega = \sqrt{2}$.

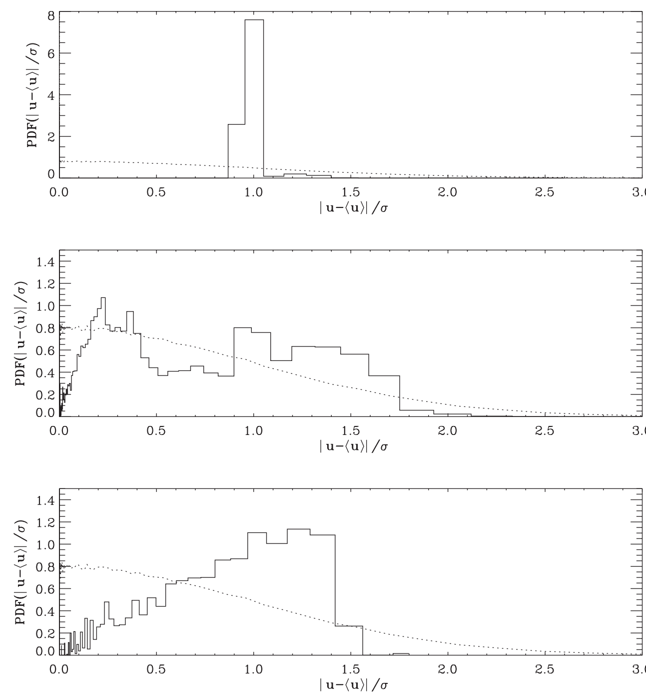

FIG. 7. Probability density functions of the absolute value of standardized velocity fluctuations of an ensemble of 10^6 particles for $\mu=0$ and $\Omega=1$. Dotted line represents the initial Maxwellian distribution, solid lines correspond to the distributions of particles at time $t_{\max}=5\times10^5$. Three different values of b_0 has been used, namely, $b_0=2\times10^{-5}$ (upper panel), $b_0=2\times10^{-6}$ (middle panel), and $b_0=2\times10^{-7}$ (lower panel).

space of the system with just one oscillating cloud is displayed, as b_0 is varied. For lower values of b_0, the phase space is characterized by the presence of Kolmogorov-Arnold-Moser (KAM) islands as in the case of the classical FUM. These curves are destroyed when b_0 increases, owing to the stochasticity of the system. When both clouds oscillate (see Fig. 4), the particle is "captured" by the clouds through a kind of resonance effect. When the parameter b_0 is nearly zero, the resonance effect is spread over a wide region in the phase space (see the top panel of Fig. 4). When the value of b_0 is higher, the particle continues to oscillate between clouds but the Fermi mechanism is, in some sense, broken: energy is neither gained nor lost anymore, both tail-on and head-on collisions work in a cooperative way. The confinement of energy of a test particle also depends on the value of μ, that is, on the fact that it can lose energy not only through tail-on collisions but also through radiative emission. The lower the value of μ the more test particles are captured through the resonance effect.

Looking at the dynamics of a single test particle, it can be easily conjectured what happens when an ensemble of test particles is put into the system of oscillating clouds: the energy confinement on each particle breaks the usual random acceleration mechanism leading to the formation of beamlets. We investigate the dynamics of the system by considering the motion of an ensemble of 10^6 particles for a given time $t_{\max}=5\times10^5$, after which we assume that the whole ensemble of particles leaves the region between the clouds. Particles are injected at random positions within the clouds separation, and initial velocities are extracted from a one–dimensional Maxwellian distribution normalized to the unitary thermal velocity

$$P(u) = \frac{1}{\sqrt{2\pi}} \exp[-u^2/2]. \tag{4}$$

In particular, we look for possible modifications of the probability distribution function (PDF) of particle velocities at time $t_{\max}$, with respect to the initial distribution. In Fig. 5 PDFs of the absolute value of the standardized velocity fluctuations $(u-\langle u\rangle)/\sigma$, collected at time $t_{\max}$, compared to the initial one, for three values of the parameter μ and for $\Omega=1$ are shown. As conjectured previously, the main feature of PDFs is the clear formation of a beamlet, say rather than a Maxwellian distribution; after a time $t_{\max}$, all particles gain more or less the same energy thus generating a quasi-monoenergetic beam. This is more visible for lower values of μ, namely, when μ increases the beam broadens and the ensemble of particles tends to become Maxwellian.

The generation of beamlets is a robust mechanism with respect to variation of parameters. A beamlet is indeed clearly formed even in the case of non rational ratio between oscillation frequencies, as shown in Fig. 6, where $\Omega=\sqrt{2}$. In this last case, at variance with the case $\Omega=1$, beamlets clearly survive even for larger values of μ (compare lower panels of Figs. 5 and 6). The generation of beamlets of energetic particles observed in Figs. 5 and 6 is also clearly recognized as the parameter b_0 is varied, as results from Fig. 7, where PDFs of the standardized velocity fluctuations at $t_{\max}$, for $\mu=0$, are displayed for three different values of b_0. The formation of one or more beamlets is clearly visible in all cases, especially when particles can penetrate a longer distance inside the clouds, that is, when b_0 is large enough.

IV. CONCLUSIONS

In the present paper, we investigated the dynamics of a model for stochastic acceleration in which nonrelativistic particles interact with two oscillating clouds with similar characteristics. Particles are also allowed to penetrate inside the clouds, the penetration depth depending on both the velocity of each particle and the magnetic field inside the cloud. The particle can also radiate energy during these periods of time. We found that, with a value of the penetration inside a cloud small enough, the system behaves similar to a standard FUM, while, increasing the penetration depth, a strong stochasticity occurs which destroys the invariant spanning curves in the phase space. In addition, when the energy radiated by a particle is either neglected or very small, well-defined high-energy beamlets are generated. Beamlets tend to be destroyed only when the radiation energy rate is high enough. This is perhaps the most interesting point of our paper, namely, the fact that an increasing stochasticity within the system, by destroying the KAM torus, pushes the particle to become "resonant" with the cloud. In this way energy is confined and beamlets can be generated.

Even though building up a specific model for the generation of beamlets in space plasmas is out of the scope of this paper, we can note that oscillations of accelerating magnetic structures in space plasmas can be easily driven by typical instability mechanisms [27] similar to, for example, the thermal instability [28] or the tearing instability [29–31]. Moreover, it is worthwhile to remark that ion beamlets inside the Earth's magnetotail can be detected at velocities of about 600–2000 km/s which are about 3–10 times the thermal speed of the ambient particles, which is typically of the order of $u_{\mathrm{th}} \simeq 200$ km/s [24–26]. Our results show that, depending on the parameter of our toy model and without the aid of an externally imposed electric field, the stochastic acceleration should be a good physical mechanism to increase the velocity of one or more beams of particles to about 3–50 times their initial thermal speed.

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